

CHERENKOV RADIATION IN A GRAVITATIONAL WAVE BACKGROUND

Alexander B. Balakin¹

Kazan State University, Kremlevskaya street 18, 420008, Kazan, Russia,

Richard Kerner²

*LPTL, Université Pierre et Marie Curie,
Tour 22, Boîte 142, 4 Place Jussieu, 75005 Paris, France,*

José P. S. Lemos³

*CENTRA, Departamento de Física, Instituto Superior Técnico,
Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal,
and
Observatório Nacional - MCT,
Rua General José Cristino 77, 20921, Rio de Janeiro, Brazil.*

Abstract A covariant criterion for the emission of Cherenkov radiation in the field of a non-linear gravitational wave is considered in the framework of exact integrable models of particle dynamics and electromagnetic wave propagation. It is shown that vacuum interacting with curvature can give rise to Cherenkov radiation. The conically shaped spatial distribution of radiation is derived and its basic properties are discussed.

¹e-mail: dulkyn@mail.ru

²e-mail: rk@ccr.jussieu.fr

³e-mail: lemos@kelvin.ist.utl.pt

1 Introduction

The phenomenon of emission of radiation stimulated by a particle moving with superluminal speed in a medium, first observed by Cherenkov and Vavilov, and theoretically explained by Tamm and Frank, is one of the cornerstones of classical electrodynamics (see, e.g., [1] and [2]). This phenomenon, called Cherenkov radiation, is the subject of many studies related to several fields of interest, from theoretically motivated ones to collider detectors in accelerators, for instance. Here we are interested in studying the production of the Cherenkov radiation in a gravitational field.

This problem can be studied through a model containing matter, the gravitational field created by this matter, test relativistic particles and test photons induced by the moving particles. The model can be characterized by a nonzero stress-energy tensor in the right-hand side of the Einstein equations, which implies a nonzero Ricci tensor, and the Cherenkov photons may travel through the material medium. This model was taken up in [3], where Cherenkov radiation was indeed predicted to be possible.

We would like to consider a second model, which contains an external gravitational field, test relativistic particles and test photons. In this model there is no material medium. Due to the external gravitational field, there is a non-vanishing Riemann curvature tensor in the background.

Now, it is known that the gravitational field itself can be considered as a sort of medium. This possibility is suggested by a sentence of the book “Classical Field Theory” of Landau and Lifshitz [4], in the problem of paragraph 90, where it is stated that the gravitational field plays a role of a medium with electric and magnetic permeabilities equal to $1/\sqrt{h}$, where h is the determinant of the three-dimensional spatial induced metric. Such analogy, of considering the gravitational field in vacuum as a medium, is useful in several instances. If the gravitational field acts as a sort of medium, one can then study the modifications induced on the speed of light in the presence of the gravitational field itself. One can try to find models, in which a massive relativistic particle is able to move faster than photons. If such models exist, then following the classical theory, one can expect that the Cherenkov effect should become possible. Thus, one can use the Cherenkov effect as a test for verifying models of interaction between the electromagnetic and gravitational fields.

If we restrict ourselves to the so-called minimal coupling between electromagnetic and gravitational fields (the covariant derivatives are based on the Christoffel symbols), we are dealing with pure vacuum. In this case the worldlines of photons are null geodesics, therefore, the velocity of the photon coincides with the universal constant c . Thus, for pure vacuum in a gravitational background one can expect that there is no possibility for the emission of Cherenkov radiation.

The introduction of a non-minimal coupling between the electromagnetic and gravitational fields in vacuum (the Lagrangian contains the Riemann tensor) leads to models which can be classified as vacuum interacting with curvature. An example of such a vacuum has been proposed by Drummond and Hathrell [5]. This

example has appeared as a result of one-loop calculations in the framework of Quantum Electrodynamics (QED) in a gravitational background and it was shown that the velocity of light can differ from c . The vacuum of Drummond and Hathrell is in a class of vacua called “non-trivial QED vacua” [6] or “modified QED vacua” [7]. A general property of a non-trivial QED vacuum is that the velocity of light differs from c . In the other words, a non-trivial QED vacuum behaves as a sort of medium, a quasi-medium, with effective refractive index $n \neq 1$. Thus, it is natural to ask the following question: is the Cherenkov effect possible in a vacuum interacting with a gravitational field? In this paper we want to consider the Cherenkov radiation in a gravitational wave (GW) background with vacuum interacting with curvature. Gravitational radiation is an example of a non-stationary field. Thus this quasi-medium (of a vacuum interacting with curvature) behaves as a nonstationary medium. As a result, as we will see, the effective refractive index depends on time.

The purpose of our paper is to show that a covariant analysis of this problem, based on exact solutions for the coupled system of a charged particle and an electromagnetic wave in a nonlinear gravitational wave background, enables one to treat it exactly in some cases. The results of our investigation show that a GW background field, quasi-media and true material media display the Cherenkov effect, while pure vacuum does not.

The paper is organized as follows. In Section 2, starting from the classical description of the Cherenkov effect in the material medium with constant refractive index, we formulate a covariant criterion and the necessary condition for the emission of Cherenkov radiation. In Section 3 we integrate exactly the equations of particle motion as well as the Maxwell equations in the GW background and apply the criterion and the necessary condition for Cherenkov radiation. In Section 4 we consider the spatial properties of Cherenkov radiation by using two examples of longitudinal and transversal particle motion. In the Appendices we obtain and briefly discuss the details of the exact solutions of Maxwell equations.

2 The Cherenkov radiation in a curved space-time: the general criterion

The Cherenkov radiation can be explained in simple terms in Minkowskian space-time (for details, see [1] and [2]). Consider a charged particle moving uniformly with velocity \mathbf{V} in a static isotropic dielectric medium with the refraction index n .

This particle induces an electromagnetic field which has the following Fourier representation:

$$\mathbf{A}(t, \mathbf{r}) = \sum_{(l)} q_{(l)}(t) \mathbf{A}_{(l)}(\mathbf{r}) + q_{(l)}^*(t) \mathbf{A}_{(l)}^*(\mathbf{r}), \quad (1)$$

where $\mathbf{A}(t, \mathbf{r})$ is the potential three-vector, $q_{(l)}(t)$ are the Fourier amplitudes with $(l) = 1, 2, \dots, \infty$. The modes $\mathbf{A}_{(l)}(\mathbf{r})$ are given by

$$\mathbf{A}_{(l)}(\mathbf{r}) = \sqrt{4\pi} \frac{c}{n} \mathbf{e}_{(l)} e^{i\mathbf{k}_{(l)} \cdot \mathbf{r}}, \quad (2)$$

where $\mathbf{k}_{(l)}$ with the fixed (l) is the wave three-vector of the mode enumerated by (l) , and $\mathbf{e}_{(l)}$ is the polarization three-vector with unit length. Then, in the Lorentz gauge $\partial_\mu A^\mu = 0$, and from Maxwell's equations [2]

$$\frac{n^2}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} - \Delta \mathbf{A} = \frac{4\pi}{c} \mathbf{j}, \quad \text{and} \quad \frac{n^2}{c^2} \frac{\partial^2 \varphi}{\partial t^2} - \Delta \varphi = 4\pi \rho, \quad (3)$$

we get, in terms of Fourier amplitudes, the following equations

$$\frac{d^2 q_{(l)}}{dt^2} + \omega_{(l)}^2 q_{(l)} = \frac{1}{c} \int \mathbf{j}(t, \mathbf{r}) \mathbf{A}_{(l)}^*(\mathbf{r}) d^3 \mathbf{r}, \quad (4)$$

where $\omega_{(l)} \equiv ck_{(l)}/n$. For a point particle with electric charge e moving with constant velocity \mathbf{V} , the electric current $\mathbf{j}(t, \mathbf{r})$ has the form

$$\mathbf{j}(t, \mathbf{r}) = e \mathbf{V} \delta(\mathbf{r} - \mathbf{V}t), \quad (5)$$

and we obtain from (4) the equation of an oscillator solicited by an external force:

$$\frac{d^2 q_{(l)}}{dt^2} + \omega_{(l)}^2 q_{(l)} = \sqrt{4\pi} \frac{e}{n} (\mathbf{e}_{(l)} \cdot \mathbf{V}) e^{-i(\mathbf{k}_{(l)} \cdot \mathbf{V})t}. \quad (6)$$

The external force in the right hand side of the equation (6) is periodic with the frequency $\Omega_{(l)} = (\mathbf{k}_{(l)} \cdot \mathbf{V}) = k_{(l)} V \cos \theta_{(l)}$, where $\theta_{(l)}$ is the angle between the direction of the wave three-vector $\mathbf{k}_{(l)}$ and the three-velocity of the particle \mathbf{V} . When

$$\Omega_{(l)} = \omega_{(l)}, \quad \text{or equivalently} \quad V \cos \theta_{(l)} = c/n, \quad (7)$$

a resonance occurs, making the oscillator's amplitude $q_{(l)}$ grow linearly with t . This resonance condition is interpreted in the book [2] as a condition for the existence of Cherenkov's radiation. Since $|\cos \theta_{(l)}| \leq 1$, one sees from (7) that the resonance is formally possible when $V \geq c/n$. Usually one excludes the case $\cos \theta_{(l)} = 1$ which corresponds to the radiation with zero aperture, and one obtains that the condition for the Cherenkov radiation emission is

$$V > \frac{c}{n}, \quad (8)$$

i.e., the velocity of the particle should be greater than the velocity of light in the medium c/n . It is also clear that for fixed V the maximal angle of radiation, the Cherenkov angle, is given by the condition

$$\cos \theta_0 = \frac{c}{nV}. \quad (9)$$

In order for this radiation to represent a real electromagnetic wave propagating in a medium with the refraction index n , the relation between the electromagnetic wave frequency ω and the modulus of the wave vector K should be $\omega = Kc/n$ [1, 2].

Considering the identifications $K \equiv k_{(l)}$ and $\theta \equiv \theta_{(l)}$, where the fixed (l) corresponds to the resonant mode, we obtain the following criterion from (7) :

$$\omega \equiv K \frac{c}{n} = KV \cos \theta = (\mathbf{K} \cdot \mathbf{V}) . \quad (10)$$

This criterion for Cherenkov's radiation emitted by a charged particle must be reformulated in a covariant way when the particle is supposed to move in a curved spacetime. In covariant formulation we cannot use the three-velocity vector \mathbf{V} , or the Cherenkov angle θ_0 , because they are not Lorentz invariants. There are two covariant vectors in this problem, the time-like momentum four-vector of the charged particle P^i , normalized according to $P_i P^i = m^2 c^2 > 0$, and the wave four-vector K_i characterizing the electromagnetic plane wave which can, in principle, propagate inside the medium in a given space-time. We say therefore that the Cherenkov radiation can exist when the following equality is satisfied :

$$K_m P^m = 0 , \quad (11)$$

where latin indices are space-time indices running from 0 to 3, and where we use the metric with signature $(+ - - -)$.

One can easily recover the relations (8)-(10). In a standard three-dimensional context in Minkowskian space-time the relationship (11) can be rewritten in the usual form

$$K_m P^m = K_0 P^0 - \mathbf{K} \cdot \mathbf{P} = \frac{P^0}{c} (\omega - \mathbf{K} \cdot \mathbf{V}) = \frac{P^0 \omega}{c} \left(1 - \frac{1}{c} \mathbf{n} \cdot \mathbf{V} \right) = 0 . \quad (12)$$

The classical definition of frequency is $\omega = cK^0$, the vector refraction index is $\mathbf{n} = c\mathbf{K}/\omega$, and its square is $n^2 = \mathbf{n}^2$. We can now reproduce the criterion to observe the Cherenkov radiation in a flat space-time, i.e., we can reproduce equations (8)-(10). For instance, using the definition of the angle θ , i.e., $\cos \theta = \mathbf{K} \cdot \mathbf{V} / KV = \mathbf{n} \cdot \mathbf{V} / nV$, as well as the condition $|\cos \theta| \leq 1$, one can see from (12) that $V \geq c/n$, i.e., particle's three-velocity should be bigger than speed of light in the medium, recovering (8). Equation (9) and (10) also follow in a straightforward way from (12).

Let us now find the necessary condition for the existence of Cherenkov's radiation. It can be given in three equivalent ways, the first involving the square $K_i K^i$ of the wave four-vector K^i , the second one involving the square n^2 of the scalar refraction index, and the third one involving the phase velocity of light in the medium, v_{ph} . Note that this necessary condition does not depend on particle's momentum P^i . It is well known that the frequency ω (considered as a scalar quantity), and the four-vector K_i^* (the spatial part of the wave four-vector) may be defined by the following relations (see, e.g., [8]) :

$$\frac{\omega}{c} \equiv K_i U^i , \quad (13)$$

and

$$K_i^* \equiv K_j \Delta_i^j . \quad (14)$$

Here U^i is the four-velocity vector of the medium (or of the observer, if we consider propagation in pure vacuum), and Δ_i^l is the projector defined as

$$\Delta_j^i \equiv \delta_j^i - U^i U_j. \quad (15)$$

Following the standard definition (see p.290 in [1]), we introduce now the four-vector refraction index n_i :

$$n_i = \frac{c}{\omega} K_i^*. \quad (16)$$

The vector refraction index n_i is in general a spacelike four-vector, orthogonal to the four-velocity U_i , and only in the special case of isotropic medium it reduces to a scalar. Its absolute value is referred to as the scalar index of refraction, (or refraction index, for simplicity). Its square is by definition given by

$$n^2 \equiv -g^{ik} n_i n_k. \quad (17)$$

We can now write the following useful identity satisfied by K^i ,

$$K_i = U_i (K_l U^l) + K_i^*. \quad (18)$$

Now, in order to use the criterion (11) we note that P^i and K^i must be orthogonal. This happens only when K_i is spacelike, since the P^i four-vector is timelike. From the equation (18) we obtain the square of K_i ,

$$K^2 \equiv -g^{im} K_i K_m = -K_m K^m = -(K^m U_m)^2 - K^l K^s \Delta_{ls}. \quad (19)$$

This equation can be written explicitly in two ways,

$$K^2 = \left(\frac{\omega}{c} \right)^2 (n^2 - 1), \quad (20)$$

and

$$K^2 = (K^*)^2 - \frac{\omega^2}{c^2}, \quad (21)$$

where

$$(K^*)^2 \equiv -K^l K^s \Delta_{ls}. \quad (22)$$

We see from the equation (20) that the sign of K^2 coincides with the sign of $(n^2 - 1)$. This means that K_i is spacelike for $n^2 > 1$. From the equation (21) it can be inferred that K_i is spacelike when $\omega^2/c^2 < (K^*)^2$, i.e. when the phase velocity of light $v_{\text{ph}} \equiv \omega/K^*$ obeys $v_{\text{ph}} < c$. We can use one of the three equivalent invariant forms of the necessary condition for existence of Cherenkov radiation, namely,

$$K_m K^m < 0, \quad \text{or,} \quad n^2 > 1, \quad \text{or,} \quad v_{\text{ph}} < c. \quad (23)$$

All of them require the knowledge of the four-vector K^i , which is obtained from the corresponding solution of Maxwell equations.

In order to use the criterion (11) in a curved background we have to resort now to a specific model. We use as a background the pp-wave solution of Einstein equations in vacuum. We can then determine the four-momentum P^i of a particle moving in this background, and find a specific solution of Maxwell equations, in the same gravitational background. After that, we can examine the criteria of existence of the Cherenkov radiation, and establish its spatial properties in this background. In a curved space-time different spatial directions are generally non-equivalent, which implies that one should know the evolution of particle's four-momentum P^i with arbitrary initial data in order to be able to use explicitly the criterion (11).

3 Charged particle in the GW background and the Cherenkov radiation

3.1 The gravitational wave background

Let us consider the space-time described by the exact pp-wave solution of Einstein's equations in vacuum [9]. The metric describing a gravitational wave propagating in the x^1 direction is supposed to take on the following form:

$$ds^2 = 2dudv - L^2 \left[e^{2\beta} (dx^2)^2 + e^{-2\beta} (dx^3)^2 \right], \quad (24)$$

where

$$u = \frac{ct - x^1}{\sqrt{2}}, \quad v = \frac{ct + x^1}{\sqrt{2}} \quad (25)$$

are the retarded and the advanced times, respectively. The functions L and β depend only on the variable u , $L = L(u)$ and $\beta = \beta(u)$.

The pp-wave metric (24) is invariant under the G_5 symmetry group and admits the following set of five Killing vector fields $\xi_{(r)}$ (where the index (r) takes on the values (v) , (2) , (3) , (4) and (5) , and characterizes each vector),

$$\begin{aligned} \xi_{(v)}^i &= \delta_v^i, & \xi_{(2)}^i &= \delta_2^i, & \xi_{(3)}^i &= \delta_3^i, \\ \xi_{(4)}^i &= x^2 \delta_v^i - \delta_2^i \int g^{22}(u) du, & \xi_{(5)}^i &= x^3 \delta_v^i - \delta_3^i \int g^{33}(u) du. \end{aligned} \quad (26)$$

Here $g^{\alpha\beta}(u)$ ($\alpha, \beta = 2, 3$) are the contravariant components of the metric tensor. The vector $\xi_{(v)}^i$ is isotropic, covariantly constant and orthogonal to the other four ones,

$$\nabla_k \xi_{(v)}^i = 0, \quad g_{ik} \xi_{(v)}^i \xi_{(r)}^k = 0. \quad (27)$$

The three vectors $\xi_{(v)}^i, \xi_{(2)}^i, \xi_{(3)}^i$ form the Abelian subgroup G_3 . The two functions $L(u), \beta(u)$ are coupled by the Einstein equation, unique in this case,

$$L'' + L (\beta')^2 = 0. \quad (28)$$

The function $\beta(u)$ can be chosen at will, and once given, one may solve the equation (28) for $L(u)$. The curvature tensor has two non-vanishing components:

$$-R^2_{.u2u} = R^3_{.u3u} = L^{-2} \left[L^2 \beta' \right]' . \quad (29)$$

Both the Ricci tensor R_{ik} and the curvature scalar R are equal to zero.

3.2 Particle dynamics in the GW background

Solving the geodesic equation for a particle with mass m in the GW field (24)

$$\frac{DP_i}{D\tau} = 0, \quad \text{with} \quad P^i = mc \frac{dx^i}{d\tau}, \quad (30)$$

and using the well-known property of the Killing vectors (26) [10], we obtain the following expressions for the components of the momentum P_i :

$$P_v \equiv P_i \xi^i_{(v)} = \text{const} \equiv C_v, \quad (31)$$

$$P_\alpha \equiv P_i \xi^i_{(\alpha)} = \text{const} \equiv C_\alpha, \quad (32)$$

$$P_u = \frac{1}{2C_v} \left[m^2 c^2 - g^{\alpha\beta} C_\alpha C_\beta \right], \quad (33)$$

where, the last equation for the component P_u of the momentum followed from the normalization condition.

Since C_v and C_α are constants, they also represent the initial values of the corresponding momentum components at the initial surface defined by $u = 0$. These data determine the character of the particle's motion. For example, when $C_\alpha = 0$, i.e., when the particle moves initially along the longitudinal direction (the direction of propagation of the GW x^1), then it will always move along that direction without acceleration. When $C_\alpha \neq 0$, the dynamical effects on the particle induced by the GW appear in its longitudinal motion (see the expression for P_u (33)). Thus, in the field of GW, the criterion (11) yields the following equation:

$$\begin{aligned} K_m P^m &= K_u P_v + K_v P_u + K_\alpha P^\alpha = \\ &= K_u C_v + K_v \frac{1}{2C_v} \left[m^2 c^2 - g^{\alpha\beta}(u) C_\alpha C_\beta \right] + g^{\alpha\beta}(u) K_\alpha C_\beta = 0. \end{aligned} \quad (34)$$

We should now represent explicitly the components of the wave four-vector K_i , and we also have to check the condition (23) necessary for the emission of the Cherenkov radiation.

3.3 Maxwell's equations in the GW background

The solutions of Maxwell's equations in a gravitational wave background have been discussed in a strong GW field in the framework of geometrical optics approximation (e.g., [11]), in the case of weak GW (see, e.g. [12] and the references therein), and in the case of strong GW without the geometrical optics approximation [13].

In this section we follow the approach presented in the article [13], in which it has been shown that the problem of propagation of electromagnetic waves in a GW background (in pure vacuum, or in vacuum interacting with curvature) is an exactly integrable problem. This approach has some advantages. For instance, if we deal with the vacuum case, it is not needed to restrict the solution either to the geometrical optics, or to the weak gravitational field approximations.

On the other hand, Maxwell's equations in a medium in presence of a gravitational wave field pose some additional problems and lead to a nonreducible system of equations. Therefore, when we deal with the Maxwell equations in a spatially isotropic medium, we have to consider the eikonal equation, i.e., we use the geometrical optics approximation (see, e.g., [8] for the description of covariant formalism). In the Appendix A we describe the methods of obtaining the solutions of Maxwell equations for pure vacuum, for vacuum interacting with curvature, for spatially isotropic dielectric medium and for spatially isotropic medium interacting with curvature. Below, we extract the information given in the Appendix A on the wave four-vector K^i , the dispersion relation $\omega(K^*)$, and the refraction index, and use it for investigating the possibility of existence of Cherenkov's radiation in a GW background for pure vacuum case and for vacuum interacting with curvature. In the Appendix B we study briefly the possibility of Cherenkov's radiation in a spatially isotropic medium interacting with curvature.

3.3.1 Pure vacuum

As it is shown in the Appendix A (see section A1.1), the wave four-vector K_i has the following form:

$$K_i = \delta_i^u \left[-\frac{k_\alpha k_\beta}{2k_v} g^{\alpha\beta}(u) \right] + \delta_i^v k_v + \delta_i^2 k_2 + \delta_i^3 k_3, \quad (35)$$

where k_v, k_2, k_3 are arbitrary constants, which, together with the constant k_1 , give the initial components of the wave four-vector. One can check easily that the square of this vector is equal to zero,

$$g^{ij} K_i K_j = 0. \quad (36)$$

Thus, the wave four-vector is isotropic and the phase velocity of the electromagnetic wave is equal to the speed of light in vacuum, yielding

$$\omega = cK^*. \quad (37)$$

From the equations (20)-(21), the scalar refraction index is given by

$$n = 1. \quad (38)$$

Therefore, the criterion (11) (or 34) can not be satisfied and there is no possibility for the Cherenkov radiation to be emitted in pure vacuum.

3.3.2 Vacuum interacting with curvature

As it is shown in section A1.2 of Appendix A, the interaction of vacuum with curvature (studied first in [5]) leads to birefringence, i.e., the wave four-vector K_i depends on polarization. From the Appendix A we have that there are two distinct electromagnetic waves, each one represented by a corresponding wave four-vector, $K_i^{(2)}$ or $K_i^{(3)}$, given by

$$K_i^{(2)} = \delta_i^u \left[-\frac{k_\alpha k_\beta}{2k_v} g^{\alpha\beta}(u) + k_v q R_{.u3u}^3 \right] + \delta_i^v k_v + \delta_i^2 k_2 + \delta_i^3 k_3, \quad (39)$$

$$K_i^{(3)} = \delta_i^u \left[-\frac{k_\alpha k_\beta}{2k_v} g^{\alpha\beta}(u) - k_v q R_{.u3u}^3 \right] + \delta_i^v k_v + \delta_i^2 k_2 + \delta_i^3 k_3, \quad (40)$$

where k_v, k_2, k_3 are again arbitrary constants, which, with k_1 , give the initial components of the wave four-vector. Introducing the term $s^{(A)} \equiv (-1)^{(A)}$ with $(A) = 2, 3$ one can condense formulae (39)-(40) into a single formula

$$K_i^{(A)} = \delta_i^u \left[-\frac{k_\alpha k_\beta}{2k_v} g^{\alpha\beta}(u) + s^{(A)} k_v q R_{.u3u}^3 \right] + \delta_i^v k_v + \delta_i^2 k_2 + \delta_i^3 k_3. \quad (41)$$

The vectors $K_i^{(2)}$ and $K_i^{(3)}$ are orthogonal to each other,

$$g^{ij} K_i^{(2)} K_j^{(3)} = 0, \quad (42)$$

and their squares are given by,

$$g^{ij} K_i^{(2)} K_j^{(2)} = 2q(k_v)^2 R_{.u3u}^3, \quad g^{ij} K_i^{(3)} K_j^{(3)} = -2q(k_v)^2 R_{.u3u}^3. \quad (43)$$

Here $R_{.u3u}^3$ is the independent component of the Riemann tensor and q is the interaction parameter of the vacuum with curvature (see Appendix A). Note that the signs in (43), depend on the sign of $q R_{.u3u}^3$, this being positive or negative at different times and in different spatial points. Since each of the two vectors $K^{(A)}$, has opposite sign, when $R_{.u3u}^3 \neq 0$, one of the two vectors has a negative square. Thus, one of the electromagnetic waves propagates with phase velocity less than the velocity of light in this vacuum interacting with curvature, and the necessary condition for Cherenkov radiation (23) is satisfied.

The frequency and the spatial part of the wave vector depend on the index (A) . Thus, using $U_i = \frac{1}{\sqrt{2}}(\delta_i^u + \delta_i^v)$ for the four-velocity of the medium (or observer) at rest, one can write from (41) the formula for the two frequencies

$$\frac{\omega^{(A)}}{c} \equiv K_i^{(A)} U^i = \frac{k_v}{\sqrt{2}} \left(1 + s^{(A)} q R_{.u3u}^3 - \frac{1}{2k_v^2} g^{\alpha\beta} k_\alpha k_\beta \right). \quad (44)$$

In a spherical representation of the initial components of the wave four-vector we can write,

$$k_1 = -k \cos \theta_0, \quad k_2 = -k \sin \theta_0 \cos \varphi_0, \quad k_3 = -k \sin \theta_0 \sin \varphi_0, \quad k_v = \sqrt{2} k \sin^2 \frac{\theta_0}{2}, \quad (45)$$

where θ_0 and φ_0 are the polar and azimuthal angles of the initial direction of the propagation of the electromagnetic wave. Note that $k_1^2 + k_2^2 + k_3^2 \equiv k^2$, and $k_v = \frac{1}{\sqrt{2}}(k_0 + k_1)$.

Now, when $\theta_0 \neq 0$, the frequencies depend on retarded time u as well as on θ_0 and φ_0 and (44) yields

$$\frac{\omega^{(A)}}{c} = k \sin^2 \frac{\theta_0}{2} \left[1 + s^{(A)} q R_{u3u}^3 + \frac{1}{L^2} \cot^2 \frac{\theta_0}{2} (\cosh 2\beta - \cos 2\varphi_0 \sinh 2\beta) \right]. \quad (46)$$

Analogously, for the two scalar indices of refraction we obtain

$$(n^{(A)})^2 = 1 - \frac{4s^{(A)} q R_{u3u}^3}{\left[1 + s^{(A)} q R_{u3u}^3 + \frac{1}{L^2} \cot^2 \frac{\theta_0}{2} (\cosh 2\beta - \cos 2\varphi_0 \sinh 2\beta) \right]^2}. \quad (47)$$

Using (20), (43) and (45) one can describe explicitly the dispersion properties of the vacuum interacting with curvature, i.e., represent the refraction index $n^{(A)}$ as a function of $\omega^{(A)}$ and k :

$$(n^{(A)})^2 = 1 - \left(\frac{c k}{\omega^{(A)}} \right)^2 4s^{(A)} q R_{u3u}^3 \sin^4 \frac{\theta_0}{2}. \quad (48)$$

We see that the refraction index depends not only on $\omega^{(A)}$ and k , but also on the polarization index (A) , on the polar angle θ_0 and on the retarded time u . Therefore, analysing the formula (48) following the lines of the book [2] one can say that the vacuum interacting with curvature behaves as a spatially anisotropic nonstationary quasi-medium, admitting birefringence and transient radiation phenomena.

In the particular case $\theta_0 = \pi$ one obtains $k^1 = -k$, i.e., the GW and the electromagnetic wave propagate in opposite directions. For the index of refraction we obtain the following simple formula :

$$n^{(A)} = \frac{1 - s^{(A)} q R_{u3u}^3}{1 + s^{(A)} q R_{u3u}^3}. \quad (49)$$

One of the two indices of refraction exceeds unity $n^{(A)} > 1$, when $s^{(A)} q R_{u3u}^3 < 0$. For a periodic gravitational radiation the term R_{u3u}^3 oscillates, so, the indices of refraction $n^{(A)}$ become periodically larger or smaller than unity. The wave four-vector periodically becomes space-like or time-like and the electromagnetic wave propagates with subluminal or superluminal phase velocity, respectively. Thus, for one of the two orthogonally polarized waves (for $(A) = 2$ or $(A) = 3$) the necessary condition for the Cherenkov radiation emission is satisfied.

In the limiting case $\theta_0 = 0$, one obtains $k_v = 0$. The wave vectors become isotropic and both of the refraction indices are equal to one, $n^{(A)} \equiv 1$. There is no Cherenkov radiation in this direction.

4 Spatial properties of the Cherenkov radiation in a GW background

Landau et al. give a problem for finding the conditions for Cherenkov radiation, as well as, the cone of its wave vectors for a particle moving uniformly in a uniaxial non-magnetic crystal, along the optical axis, and at right angles to it ([1], p.407). In such a crystal there is birefringence, i.e., the wave four-vector K_i and the velocity of the wave depend on the polarization. Usually, there are two waves, the ordinary and the extraordinary, propagating with different speeds. In this problem of Landau et al., it is found that along the optical axis the radiation is contained in a circular cone, whereas at right angles to it there are two cones, corresponding to the ordinary and extraordinary waves, the one for the ordinary wave being circular, and the other for the extraordinary wave being a cone with different geometry, a non-circular one.

We will see now that in our problem there are two privileged directions along which one can find the geometry of the cones of the Cherenkov radiation. Indeed, the GW background defines the direction of propagation of the wave, the longitudinal direction, and the directions orthogonal to it, i.e. the transversal ones.

In order to describe the spatial properties of Cherenkov radiation, we study the criterion (34) for the case of vacuum interacting with curvature (see section 3.3.2, for another case see Appendix B). For this model the exact solutions for the wave four-vectors, $K_i^{(2)}$ and $K_i^{(3)}$, are given in equations (39)-(40). Then equation (34) has the form

$$\left(-\frac{1}{2k_v}g^{\alpha\beta}k_\alpha k_\beta + k_v s^{(A)}qR_{u3u}^3\right)C_v + k_v \frac{1}{2C_v} \left(m^2 c^2 - g^{\alpha\beta}C_\alpha C_\beta\right) + g^{\alpha\beta}k_\alpha C_\beta = 0. \quad (50)$$

Let us now consider two cases of particle motion, one motion being along the longitudinal direction, the other along one particular transversal direction.

4.1 Longitudinal particle motion

This case describes the situation of a particle moving in a direction parallel to the x^1 axis along which the GW propagates. In this case the initial transversal components of the particle momentum vanish, i.e.,

$$C_\alpha = 0, \quad (51)$$

and the equation (50) is simplified,

$$\left(-\frac{1}{2k_v}g^{\alpha\beta}k_\alpha k_\beta + k_v s^{(A)}qR_{u3u}^3\right)C_v + k_v \frac{m^2 c^2}{2C_v} = 0. \quad (52)$$

From the definition $P^1 = \frac{1}{\sqrt{2}}(P_u - P_v)$, from the equation $P^1 = mV/\sqrt{1 - V^2/c^2}$ (where V is the particle three-velocity), and from equation (33) (with $C_\alpha = 0$) we

find,

$$\frac{\sqrt{2}C_v}{mc} = \sqrt{\frac{1 - \frac{V}{c}}{1 + \frac{V}{c}}}. \quad (53)$$

One can then rewrite equation (52) in a form displaying explicitly the dependence between the particle three-velocity V and the angles θ_0 and φ_0 as,

$$\frac{1 - \frac{V}{c}}{1 + \frac{V}{c}} + \cot^2 \frac{\theta_0}{2} (\cosh 2\beta - \cos 2\varphi_0 \sinh 2\beta) + s^{(A)} q L^2 R_{.u3u}^3 = 0. \quad (54)$$

As it was mentioned at the end of last section, for $\theta_0 = 0$, i.e., the particle and the GW propagate in the same direction, the necessary condition for the emission of Cherenkov's radiation is not satisfied.

Thus, we have to consider the particle moving in an opposite direction to the GW, i.e. $\theta_0 = \pi$. Since $V < c$ we have from equation (54) that the Cherenkov radiation exists when the following inequality holds,

$$\cot^2 \frac{\theta_0}{2} (\cosh 2\beta - \cos 2\varphi_0 \sinh 2\beta) < -s^{(A)} q L^2 R_{.u3u}^3. \quad (55)$$

This inequality (55) describes the interior of the limit cone obtained by putting $V = c$ in equation (54), i.e.,

$$\cot^2 \frac{\theta_0}{2} (\cosh 2\beta - \cos 2\varphi_0 \sinh 2\beta) = -s^{(A)} q L^2 R_{.u3u}^3. \quad (56)$$

This equation defines a non-circular cone, depending not only on the azimuthal angle φ_0 but also on the retarded time u , via $\beta(u)$ and $L(u)$. The central axis of this cone coincides with the direction $\theta_0 = \pi$. When $\beta(u)$ is negative, the instantaneous aperture of this cone is maximal at $\varphi_0 = \frac{\pi}{2}$, and is minimal for $\varphi_0 = 0$.

Looking at equation (55) we find that the left-hand side of the inequality is always positive. Therefore, a necessary condition for the Cherenkov radiation is that $s^{(A)} q R_{.u3u}^3$ be negative. This cannot happen for both polarizations at the same time, but may happen alternately for each polarization. We deal here with an interesting phenomenon within the Cherenkov radiation, namely, polarized radiation with oscillating polarization direction.

4.2 Transversal particle motion

Let us consider now the following initial data for the particle motion,

$$C_2 = 0, \quad C_3 = -\frac{mV}{\sqrt{1 - \frac{V^2}{c^2}}}, \quad C_v = \frac{mc}{\sqrt{2}\sqrt{1 - \frac{V^2}{c^2}}}, \quad (57)$$

corresponding to transversal motion to the direction of the GW propagation, i.e., motion of the particle with three-velocity V in x^3 direction on the wave front. The

criterion (34) gives in terms of θ_0 , φ_0 , and V the following equation

$$\begin{aligned} \frac{1}{L^2} \cot^2 \frac{\theta_0}{2} (\cosh 2\beta - \cos 2\varphi_0 \sinh 2\beta) + s^{(A)} q R_{.u3u}^3 = \\ = -1 + \frac{V^2}{c^2} - \frac{V}{c} \frac{1}{L^2} e^{2\beta} \left(\frac{V}{c} - 2 \cot \frac{\theta_0}{2} \sin \varphi_0 \right). \end{aligned} \quad (58)$$

For a given V , this equation describes the cone along which the Cherenkov radiation can propagate. Since it depends on φ_0 it is a non-circular cone. The aperture depends on the retarded time u , as in the previous case. The main axis of this cone is characterized by the polar angle $\pi/2$ and azimuthal angle $\pi/2$ (i.e., the x^3 -direction). For an ultrarelativistic particle $V \rightarrow c$, and in the section $\varphi_0 = \frac{\pi}{2}$, equation (58) can be reduced to

$$\left(\cot \frac{\theta_0}{2} - 1 \right)^2 = -s^{(A)} q R_{.u3u}^3 L^2 e^{-2\beta}. \quad (59)$$

Thus, the angle of emission of the created photon is predetermined by the quantity $s^{(A)} q R_{.u3u}^3 L^2 e^{-2\beta}$. Since this quantity is rather small, we deduce that this angle is very near to the direction of the particle motion given by the angle $\pi/2$. Of course, Cherenkov radiation exists, when $s^{(A)} q R_{.u3u}^3$ is negative, and this is possible for one of the two waves only.

5 Conclusions

We have seen that although a plane gravitational wave modifies the dielectric properties of pure vacuum, it does not modify its scalar refraction index, and therefore there is no possibility of emission of Cherenkov radiation in this particular case.

On the other hand, vacuum interacting with curvature, considered as a sort of quasi-medium, allows the possibility of the existence of Cherenkov radiation. We have shown that Cherenkov radiation in a vacuum interacting with curvature can propagate along a noncircular cone, the spatial structure of this cone being permanently modified with time. In addition, the Cherenkov radiation exists alternatively for each polarization of the electromagnetic wave.

The dispersion relation for vacuum interacting with curvature is shown to be analogous to a spatially anisotropic nonstationary medium. This analogy allows one to explain the appearance of the Cherenkov radiation in the GW field in terms of a transient radiation, first introduced by Ginzburg.

We have shown a new effect, namely, the existence of polarized radiation with oscillating polarization direction, where the Cherenkov angle is predetermined by the value of the Riemann tensor.

We have performed an exact treatment in two particular cases, for two privileged directions of the particle motion, the longitudinal and the transversal ones. For arbitrary direction of particle motion, the structure and the inclination of the cone's axis with respect to the GW front plane is given by more complicated modified expressions.

Now, we have used, in section 2, Ginzburg's way of explaining physically the origin of the Cherenkov radiation for a medium with constant refraction index n . However, as was shown in subsubsection 3.3.2. (see, e.g., the formula (48)), the effective refraction index for a vacuum interacting with curvature is a function of the retarded time u . Since n is not a constant one can then ask whether the conclusion about Cherenkov radiation remains valid or not. The answer is positive. To confirm this conclusion one has to analyze equations (70) supplemented by the electric current produced by the relativistic particle. One can then obtain an exact solution of the corresponding modified Maxwell equation and study it along the line of equations (3)-(6). We have performed a preliminary calculation that fully supports this conclusion. This preliminary calculation also shows that in the framework of the geometrical optics approximation, when the wavelength of GW is much larger than the wavelength of the induced electromagnetic wave, we recover the results we have obtained here for the structure of the Cherenkov light-cone. We hope to discuss this problem in detail in a forthcoming paper.

Acknowledgments

This work has been supported by CNRS (France) during the invited stay of A.B. and of J.P.S.L. in the Laboratoire G.C.R. at the University Paris-VI. A.B. expresses his thanks to the CENTRA/IST in Lisbon for its hospitality, and acknowledges a special grant for invited scientists from the Fundação para a Ciência e Tecnologia, through an ESO programme.

Appendix A. Solutions of Maxwell's equations in the GW background

A1. Exact solution of Maxwell equations for vacuum in the GW background

A1.1 Pure vacuum

Let us recall briefly the main steps needed for solving Maxwell's equations

$$\nabla_k H^{ik} = 0, \quad (60)$$

$$\nabla_k F^{*ik} = 0, \quad (61)$$

where F^{ik} is the Maxwell tensor, F^{*ik} its dual, and H^{ik} is the induction tensor. The set of equations (60) and (61) of covariant electrodynamics of continuous media should be completed by a consistent formulation of the constitutive equations [14], relating the tensor of electric and magnetic induction H^{ik} with the Maxwell tensor. The simplest relation between these tensors is linear and has the following form [14]:

$$H^{ik} = C^{ikmn} F_{mn}, \quad (62)$$

where C^{ikmn} is the material tensor, describing the properties of linear response and containing the information about dielectric and magnetic permeabilities, as well as about the magneto-electric coefficients. In order to solve Maxwell's equations it is better to find a solution for the four-vector electromagnetic potential A^i , defined as

$$F_{ik} = \nabla_i A_k - \nabla_k A_i. \quad (63)$$

Working in the Lorentz gauge,

$$\nabla_k \nabla^k A^i = 0, \quad (64)$$

then projecting the four-vector A_i into the Killing directions, $\xi_{(v)}^i, \xi_{(2)}^i, \xi_{(3)}^i$, and after some manipulation, one finds the following solution (see [13] for details),

$$A_{(2)} = e^\beta B_{(2)}(W), \quad A_{(3)} = e^{-\beta} B_{(3)}(W), \quad (65)$$

where $B_{(2)}(W)$ and $B_{(3)}(W)$ are arbitrary functions of the argument W , which in turn is given by

$$W = k_v v + k_2 x^2 + k_3 x^3 + \Phi(u). \quad (66)$$

Here k_v, k_2, k_3 are arbitrary constants, and $\Phi(u)$ is obtained from the equation:

$$2k_v \Phi'(u) + g^{\alpha\beta} k_\alpha k_\beta = 0. \quad (67)$$

When $k_v \neq 0$, we find

$$\Phi(u) = \Phi(0) - \frac{k_\alpha k_\beta}{2k_v} \int_0^u du g^{\alpha\beta}(u). \quad (68)$$

The component A_u is equal to

$$A_u = \frac{1}{k_v L^2} \left[e^{-\beta} k_2 B_{(2)}(W) + e^{\beta} k_3 B_{(3)}(W) \right]. \quad (69)$$

If $k_v \equiv 0$, we obtain from (67) $k_2 = k_3 = 0$ and the function $\Phi(u)$ is totally arbitrary. The component A_v does not enter the Maxwell tensor and can be thus set to zero.

A1.2 Vacuum interacting with curvature

Taking into account now QED corrections [5], one can find that the induction and Maxwell tensors are connected by the relationship $H^{ik} = F^{ik} + q R^{ikmn} F_{mn}$, where $q = \alpha \lambda_e^2 / 90\pi$ (α is the fine structure constant, and λ_e is the Compton wavelength of the electron). Thus, we can rewrite the Maxwell equations in the form:

$$\nabla_k \nabla^k A^i = q R^{ikmn} \nabla_k [\nabla_m A_n - \nabla_n A_m]. \quad (70)$$

This is the appropriate quantum field theoretical modification of the equation (64).

The solutions of Maxwell equation are given in this case by [13]

$$A_{(2)} = e^{\beta} B_{(2)}(W_{(2)}) \quad A_{(3)} = e^{-\beta} B_{(3)}(W_{(3)}), \quad (71)$$

where the phases $W_{(2)}$ and $W_{(3)}$ are now different from each other,

$$W_{(2)} = W + k_v q \int_0^u d\tau R_{u3u}^3(\tau), \quad W_{(3)} = W - k_v q \int_0^u d\tau R_{u3u}^3(\tau), \quad (72)$$

and the W scalar is the same as in (66). The A_u component is now given by

$$A_u = \frac{1}{k_v L^2} \left[e^{-\beta} k_2 B_{(2)}(W_{(2)}) + e^{\beta} k_3 B_{(3)}(W_{(3)}) \right]. \quad (73)$$

The component $A_{(v)}$ does not contain any curvature contribution. Therefore, it coincides with the equation for pure vacuum, and again we can put $A_{(v)} = 0$.

As an addendum, note that if $k_v = 0$, an arbitrary function of the retarded time satisfies the equations (70) automatically. In other words, the electromagnetic waves propagate in the same direction as the gravitational wave, and ignore the interactions with curvature.

A2. Solutions of Maxwell's equations in a spatially isotropic medium (electromagnetic waves in the geometrical optics approximation)

The set of equations (60) and (61) of covariant electrodynamics of continuous media with linear constitutive equation (62) can be specified for the model under consideration in the following way. When the medium, prior to the GW appearance, is isotropic and homogeneous, the tensor C^{iklm} has the following structure [15]

$$C^{ikmn} = C_{(\text{isotr})}^{ikmn} + C_{(\text{anisotr})}^{ikmn}. \quad (74)$$

The standard isotropic part $C_{(\text{isotr})}^{ikmn}$ of the tensor C^{ikmn} is given by

$$C_{(\text{isotr})}^{ikmn} \equiv \frac{1}{2\mu} (g^{im}g^{kn} - g^{in}g^{km}) + \left(\frac{\varepsilon\mu-1}{2\mu}\right) (g^{im}U^kU^n - g^{in}U^kU^m + g^{kn}U^iU^m - g^{km}U^iU^n), \quad (75)$$

with dielectric permeability ε and magnetic permeability μ , and the anisotropic contributions, linear in the Riemann tensor, are given by

$$C_{(\text{aniso})}^{ikmn} = QR^{ikmn} + \bar{Q}U_pU_q (R^{ipmq}g^{kn} - R^{ipnq}g^{km} + R^{kpnq}g^{im} - R^{kpmq}g^{in}) + \hat{Q}U_pU_q (R^{ipmq}U^kU^n - R^{ipnq}U^kU^m + R^{kpnq}U^iU^m - R^{kpmq}U^iU^n). \quad (76)$$

In the leading order approximation of geometrical optics [8] it is a common practice to represent the vector potential through a phase scalar Ψ and a slowly varying amplitude a_j defined by,

$$A_j = a_j e^{i\Psi}, \quad K_j \equiv \nabla_j \Psi, \quad (77)$$

where, K_j is a wave-four vector. Assuming that the derivative of the phase is much bigger than the derivative of the amplitude, we obtain for the Maxwell tensor

$$F_{mn} = i(K_m a_n - K_n a_m) e^{i\Psi}. \quad (78)$$

The Maxwell equation can then be reduced to

$$K_l K_n C^{ilmn} a_m = 0. \quad (79)$$

Let us now consider the equation (79) in the case of pure spatially isotropic medium, and then in the case of a medium interacting with curvature.

A2.1 Pure spatially isotropic medium

In the absence of curvature induced effects, it is convenient to introduce the generalized Lorentz condition

$$\nabla_j (A^j + (\varepsilon\mu - 1)U^j U_l A^l) = 0, \quad (80)$$

which coincides with standard Lorentz condition in vacuum ($\varepsilon = \mu = 1$), and when the vector potential A_i is orthogonal to the velocity four-vector U^i . In the eikonal approximation equation (80) yields

$$K_m a^m = (1 - \varepsilon\mu) K_l U^l a_n U^n. \quad (81)$$

Substituting the expression (75) into equation (79), one obtains, from equation (81):

$$(a^i + U^i(\varepsilon\mu - 1)a_m U^m) [K_m K^m + (\varepsilon\mu - 1)(K_m U^m)^2] = 0. \quad (82)$$

A nontrivial a^i vector exists if and only if the following relation takes place,

$$K_m K^m = (1 - \varepsilon\mu)(K_m U^m)^2. \quad (83)$$

So, comparing (83) and (20), we can conclude, that the index of refraction n coincides identically with the index of refraction of the medium $\sqrt{\varepsilon\mu}$, i.e., the gravitational wave field does not modify the index of refraction of a spatially isotropic medium. Cherenkov radiation is possible for a supraluminal moving particle, since for $\varepsilon\mu > 1$ the necessary condition is satisfied, with or without a gravitational wave field.

A2.2. Spatially isotropic medium interacting with curvature

Let us consider the particular case, of a transverse electromagnetic wave propagating parallelly to the gravitational wave, i.e., $K_\alpha = 0$ and $a_u = a_v = 0$. As a consequence, we see, that $K_m a^m = 0$. Therefore, the generalized Lorentz condition (80) leads to the relation $a_m U^m = 0$.

Taking into account formula (81), one can transform equation (79) with C^{iklm} given by (74)-(76) into the following one

$$\begin{aligned} a^i [K_m K^m + (\varepsilon\mu - 1)(K_m U^m)^2] = \\ -\mu a^\alpha R_{u\alpha u}^i [2Q(K_v)^2 + \bar{Q}K_m K^m + \hat{Q}(K_m U^m)^2]. \end{aligned} \quad (84)$$

It includes three variables K_v , $K_m K^m$ and $K_m U^m$. Using the relationships

$$2K_u K_v = K_m K^m, \quad \frac{1}{\sqrt{2}}(K_u + K_v) = K_m U^m, \quad (85)$$

one can express the first variable K_v in terms of the scalar index of refraction n , and the other two variables, $K_m K^m$ and $K_m U^m$, through the relation,

$$K_v = \frac{1}{\sqrt{2}}K_m U^m \left(1 \pm \sqrt{1 - \frac{K_m K^m}{(K_m U^m)^2}} \right) = \frac{1}{\sqrt{2}}K_m U^m (1 \pm n). \quad (86)$$

Considering equation (84) for $i = u, v$, we obtain a trivial equality. The nontrivial solutions for $a^{(A)}$ ($(A) = 2, 3$) do exist when $n = n^{(A)}$ satisfies the following quadratic equation,

$$\begin{aligned} (n^{(A)})^2 [1 - \mu s^{(A)} R_{u3u}^3 (\bar{Q} - Q)] \pm 2n^{(A)} \mu s^{(A)} R_{u3u}^3 Q - \\ - \mu [\varepsilon - s^{(A)} R_{u3u}^3 (Q + \bar{Q} + \hat{Q})] = 0. \end{aligned} \quad (87)$$

The medium becomes anisotropic due to curvature interactions, the index of refraction depends on the polarization and, consequently, birefringence appears in the same way as in the vacuum case.

Appendix B. Another case admitting Cherenkov radiation: Spatially isotropic medium interacting with curvature

As it is shown in Appendix A (see section A2.2), birefringence appears in the case of vacuum interacting with curvature. We obtain for the indices of refraction the following expressions :

$$n^{(A)} = \left[1 - \mu s^{(A)} R_{u3u}^3 (\bar{Q} - Q) \right]^{-1} \left\{ \mp \mu s^{(A)} R_{u3u}^3 Q + \sqrt{\mu^2 (R_{u3u}^3)^2 Q^2 + \mu \left[\varepsilon - s^{(A)} R_{u3u}^3 (Q + \bar{Q} + \hat{Q}) \right] \left[1 - \mu s^{(A)} R_{u3u}^3 (\bar{Q} - Q) \right]} \right\} \quad (88)$$

The sign plus in front of the square route was thus chosen because in the absence of GW the solution (88) have to coincide with $\sqrt{\mu\varepsilon}$. In the GW field the indices of refraction $n^{(A)}$ can be in general larger and smaller than the index of refraction in the medium $\sqrt{\varepsilon\mu}$. The index of refraction, being a function of the Riemann tensor component R_{u3u}^3 , changes periodically. Thus, at least for one of the electromagnetic waves with orthogonal polarization the necessary condition for the Cherenkov radiation is satisfied. When the scalars μ and ε are close to one the value of Riemann tensor predetermines the phenomenon. Depending on the case, one or both waves can propagate with subluminal phase velocity. Thus, either both waves or only one of them can describe a wave emitted due to the Cherenkov effect.

References

- [1] L. D. Landau, E. M. Lifchitz, L. P. Pitaevskii, *Electrodynamics of Continuous Media* (Butterworth Heinemann, Oxford 1996).
- [2] V. L. Ginzburg, *Physique Théorique et Astrophysique* (in French), Ed. “Mir”, Moscow, 1978.
- [3] A. Gupta, S. Mohanty, M.K. Samal, *Class.& Quantum Gravity* **16** (1999) 291.
- [4] L. D. Landau, E. M. Lifchitz, *Classical Theory of Fields* (Butterworth Heinemann, Oxford 1996).
- [5] I. T. Drummond, S. J. Hathrell, *Phys. Rev. D* **22** (1980) 343.
- [6] W. Dittrich, H.Gies, *Phys. Rev. D.* **58** (1998) 025004.
- [7] K. Scharnhorst, *Annalen Phys.* **7** (1998) 700.
- [8] J. L. Synge *Relativity: the general theory*, (Amsterdam, North Holland, 1966).
- [9] D. Kramer, H. Stephani, M. McCallum, E. Herlt, E.Schmutzer, *Exact solutions of the Einstein field equations* (Cambridge University Press, Cambridge 1980).
- [10] C. W. Misner, K. S. Thorne, J. A. Wheeler, *Gravitation*, (Freeman, San Francisco 1973).
- [11] Yu. G. Sbytov, *JETP* **71** (1976) 2001.
- [12] M. Calura, E. Montanari, *Class. & Quantum Gravity* 1998, **16**(1998) 643.
- [13] A. B. Balakin, *Class. & Quantum Gravity* **14** (1997) 2881.
- [14] G. A Maugin, *J. Math. Phys.* **19** (1978) 1198.
- [15] A. B. Balakin, J. P. S. Lemos, *Class. & Quantum Gravity* **18** (2001) 941.